POOR'S MAN MAJORANA MODES WITH SUPERCONDUCTING PHASE CONTROL

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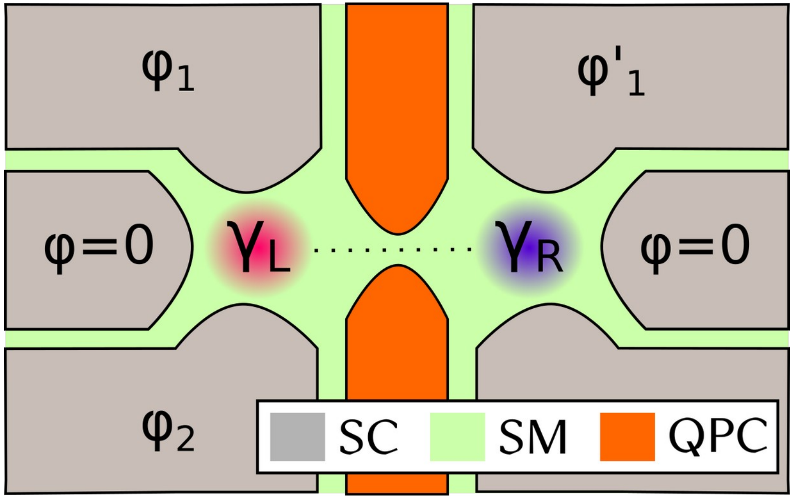
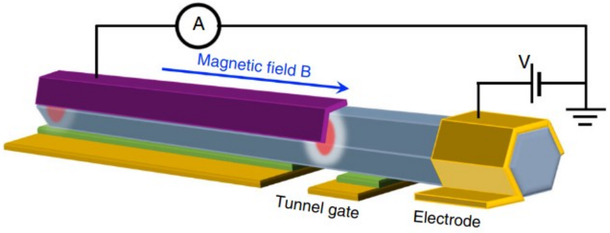
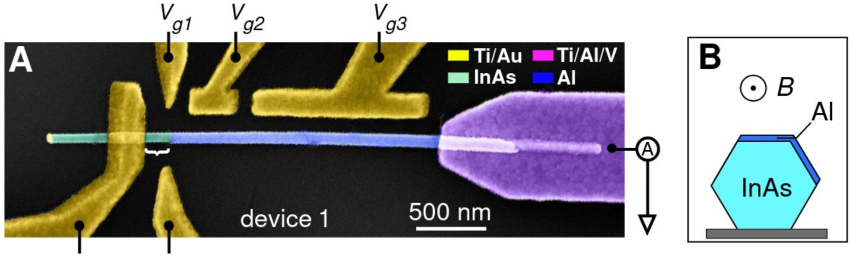


Motivation

Majorana modes are the basis of a fault-tolerant computer.

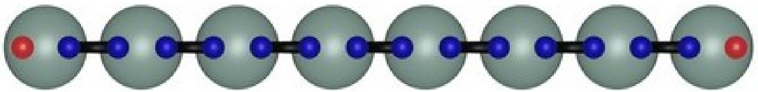
Top-bottom approach

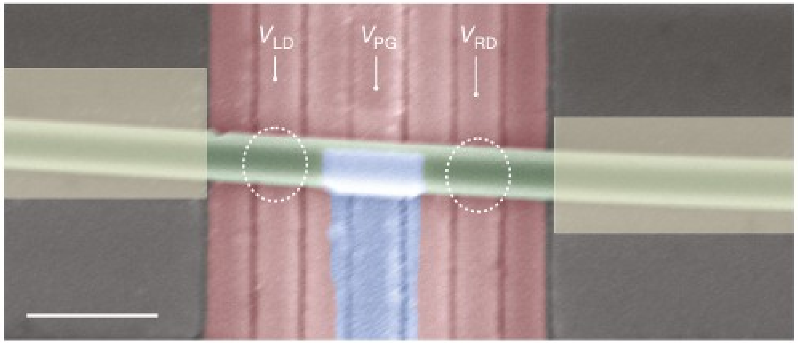




Not predictable/reproducible because of disorder

Bottom-up approach

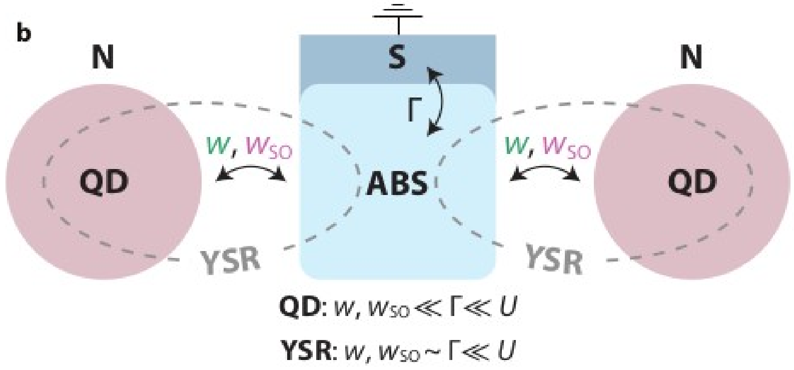




Fine tuned, but fully-controllable

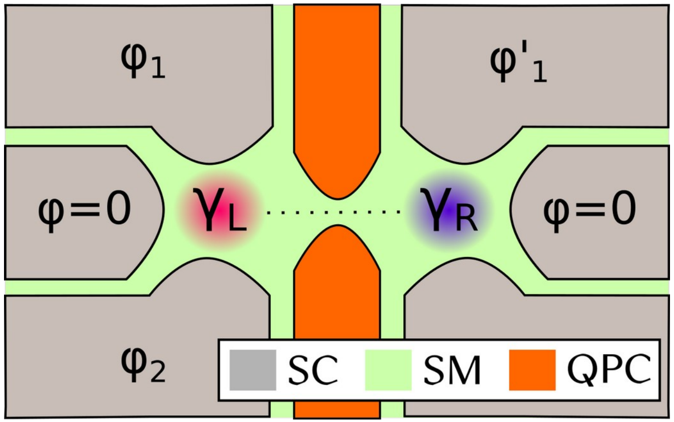
Motivation

QD vs YSR states



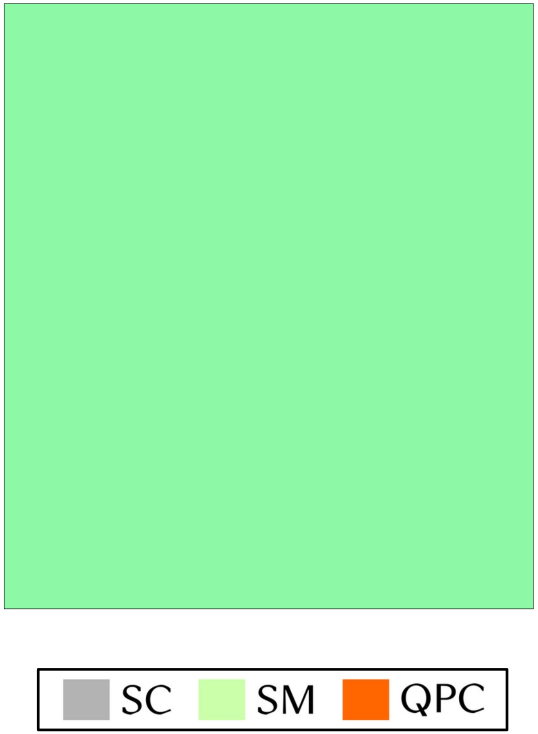
	Minigap
QD: T. Dvir et al., Nature 614, 445 (2023).	**30 μeV**
YSR: F. Zatelli et al, arXiv:2311.03193 (2023).	**70 μeV**

Josephson Junction (JJ) states



Less sensitive to charge perturbations
Larger minigaps
No magnetic fields → less decoherence

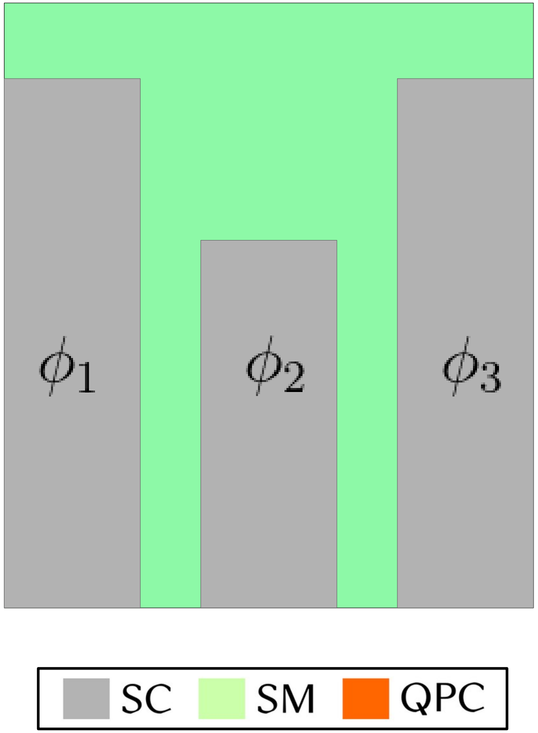
Model



$$H = \left(\frac{\hbar^2 k^2}{2m^*} - \mu\right)\sigma_0\tau_z + \vec{\alpha}\cdot\left(\vec{k}\times\vec{\sigma}\right)$$

2DEG with SOC, e.g., InAs or Ge, doesn't matter!

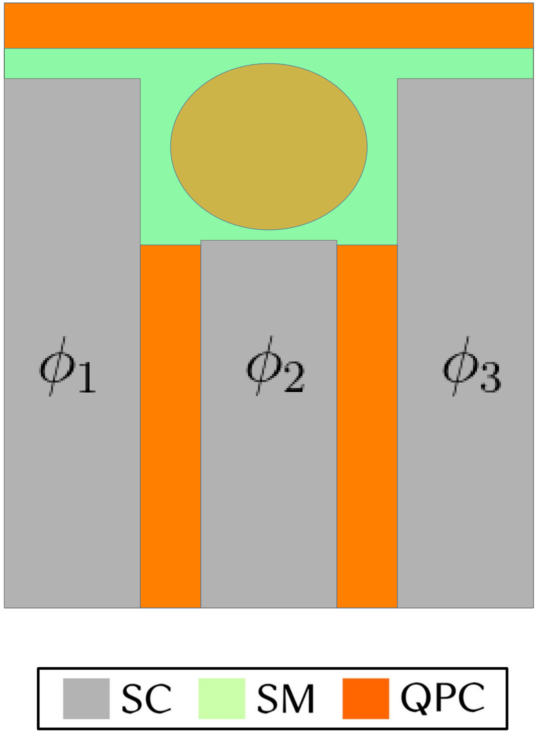
Model



$$H = \left(\frac{\hbar^2 k^2}{2m^*} - \mu\right)\sigma_0\tau_z + \vec{\alpha}\cdot\left(\vec{k}\times\vec{\sigma}\right)$$
$$+\Delta(\vec{r})\sigma_0\left(\cos\phi(\vec{r})\tau_x + i\sin\phi(\vec{r})\tau_y\right)$$

Three SCs with different SC phases.

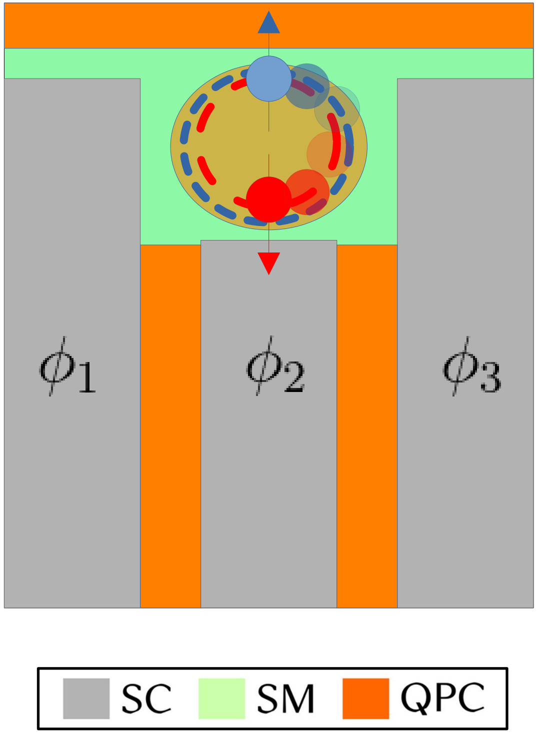
Model



$$H = \left(\frac{\hbar^2 k^2}{2m^*} - \mu\right)\sigma_0\tau_z + \vec{\alpha}\cdot\left(\vec{k}\times\vec{\sigma}\right) - e\Phi(\vec{r})\sigma_0\tau_z$$
$$+\Delta(\vec{r})\sigma_0(\cos\phi(\vec{r})\tau_x + \sin\phi(\vec{r})\tau_y)$$

Potential of the junction can be tuned.

Model

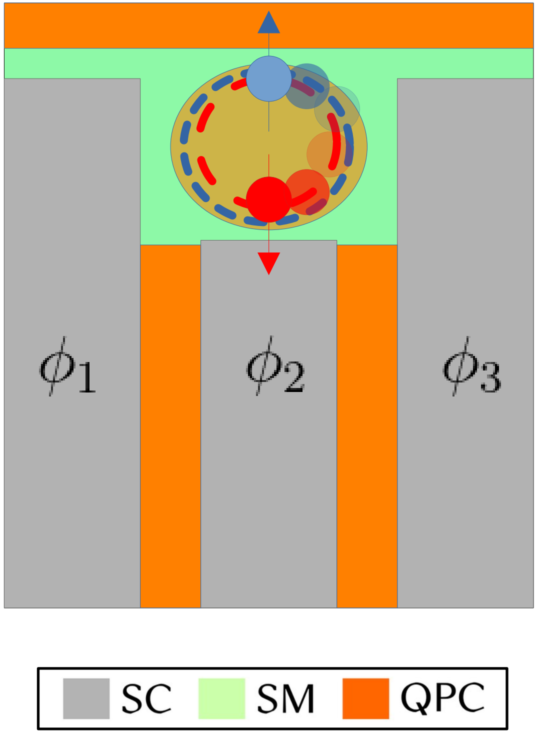


$$H = \left(\frac{\hbar^2 k^2}{2m^*} - \mu\right)\sigma_0\tau_z + \vec{\alpha}\cdot\left(\vec{k}\times\vec{\sigma}\right) - e\Phi(\vec{r})\sigma_0\tau_z$$

$$+\Delta(\vec{r})\sigma_0(\cos\phi(\vec{r})\tau_x + \sin\phi(\vec{r})\tau_y)$$

TRS can be broken due to the Aharonov–Casher effect + phase winding

Model

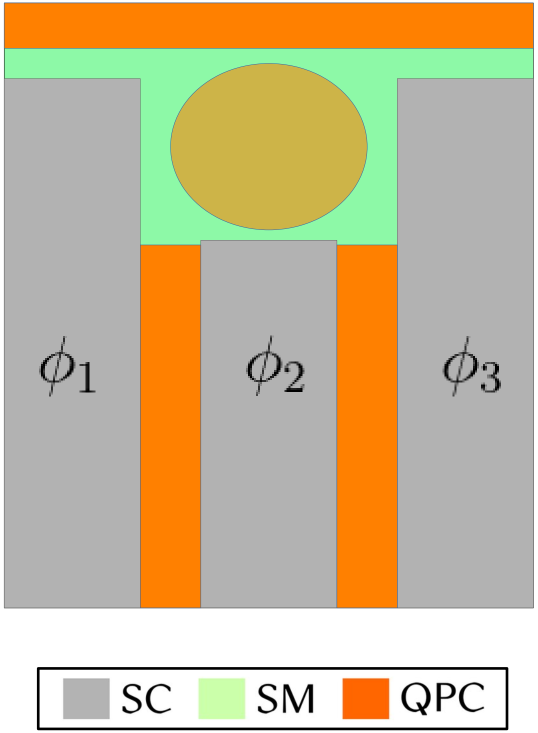


$$H = \left(\frac{\hbar^2 k^2}{2m^*} - \mu\right)\sigma_0\tau_z + \vec{\alpha}\cdot\left(\vec{k}\times\vec{\sigma}\right) - e\Phi(\vec{r})\sigma_0\tau_z$$
$$+\Delta(\vec{r})\sigma_0(\cos\phi(\vec{r})\tau_x + \sin\phi(\vec{r})\tau_y)$$

TRS can be broken due to the Aharonov–Casher effect + phase winding

The quantized (tight-binding) model must retain these ingredients

Model



$$H = H_{QD} + \sum_{j=1,2,3} H_{SC,j} + H_{t,j}$$

$$H_{QD} = \sum_{\sigma,n} \epsilon_n c^\dagger_{n,\sigma} c_{n,\sigma} + \sum_n c^\dagger_n \vec{t}_0 \cdot \vec{\sigma} c_{n+1} + h.c.$$

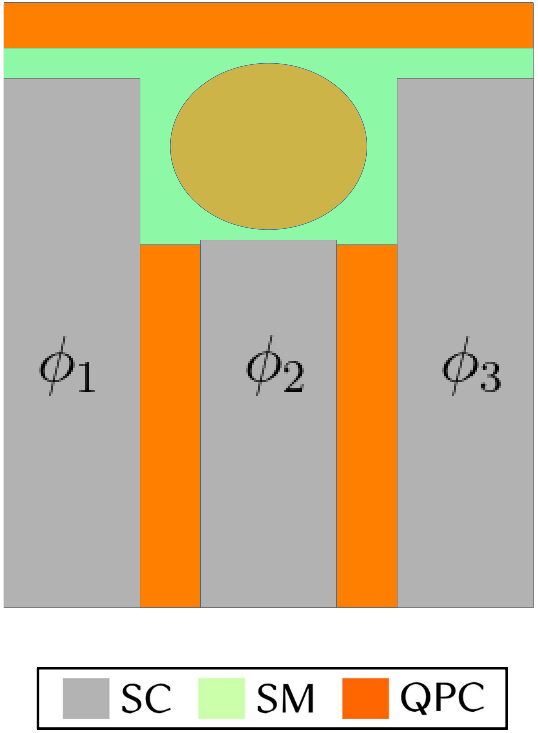
$$H_{\mathrm{SC,j}} = \sum_{\sigma,k} \epsilon_{\sigma,k} d^\dagger_{j,\sigma,k} d_{j,\sigma,k} + \Delta e^{i\phi_j} d_{j,\sigma,k} d_{j\sigma,k} + h.c.,$$

$$H_{t,j} = \sum_{k,n} t_j d^\dagger_{j,k} U_j \chi_{j,n} c_n$$

$$U_j = \cos(k_{SO}R)\sigma_0 + i\sin(k_{SO}R)\left[\sin(\theta_j)\sigma_x - \cos(\theta_j)\sigma_y\right]$$

$$c_\sigma(\vec{r}) = \sum_n \chi(\vec{r}) c_{n,\sigma}$$

Model



$$H = H_{QD} + \sum_{j=1,2,3} H_{SC,j} + H_{t,j}$$

We take n={1,2}

$$H_{QD} = \sum_{\sigma,n} \epsilon_n c^\dagger_{n,\sigma} c_{n,\sigma} + \sum_n c^\dagger_n \vec{t}_0 \cdot \vec{\sigma} c_{n+1} + h.c.$$

$$H_{\mathrm{SC,j}} = \sum_{\sigma,k} \epsilon_{\sigma,k} d^\dagger_{j,\sigma,k} d_{j,\sigma,k} + \Delta e^{i\phi_j} d_{j,\sigma,k} d_{j\sigma,k} + h.c.,$$

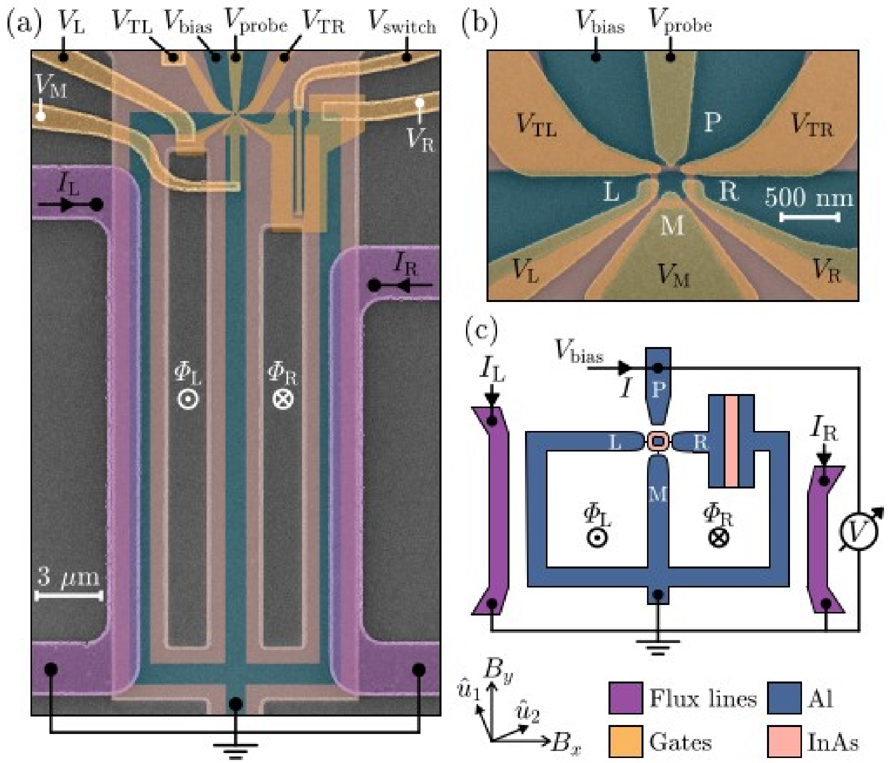
Everything depends on $k_{SO}R$

$$H_{t,j} = \sum_{k,n} t_j d^\dagger_{j,k} U_j \chi_{j,n} c_n$$

$$U_j = \cos(k_{SO}R)\sigma_0 + i\sin(k_{SO}R)\left[\sin(\theta_j)\sigma_x - \cos(\theta_j)\sigma_y\right]$$

$$c_\sigma(\vec{r}) = \sum_n \chi(\vec{r}) c_{n,\sigma}$$

Model



M. Cariola et al., Nat. Com. 4, 6784 (2023).
M. Cariola et al, 2307.06715 (2023).

We use the parameters that *fits the best* their exp. data

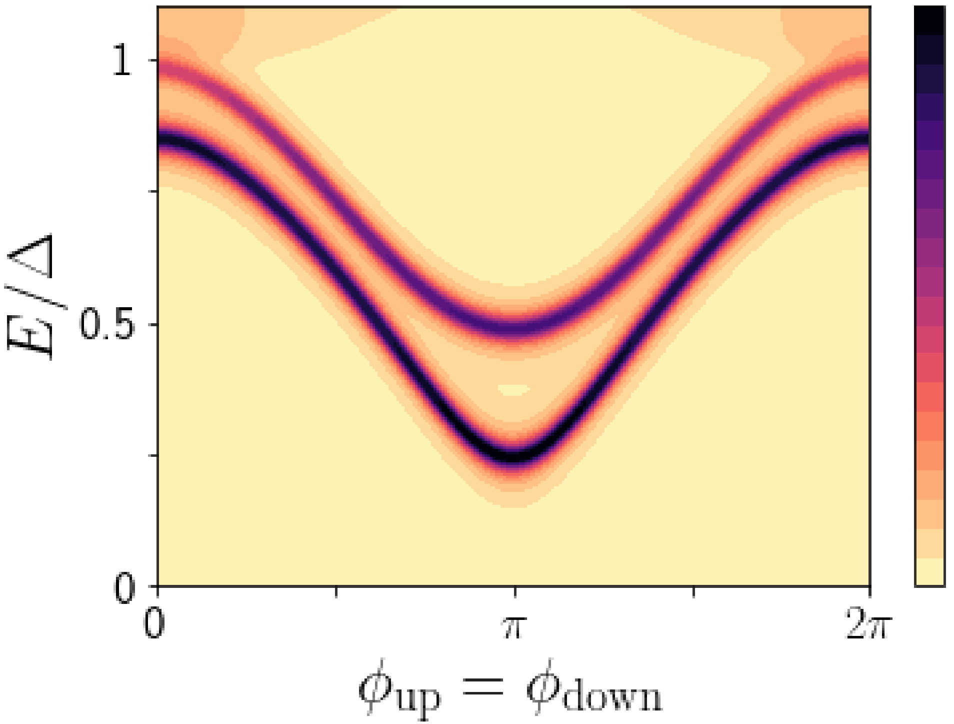
$$\Delta = 0.2 \text{ meV} \qquad \Gamma \simeq 6\Delta \qquad t_0 \simeq \Delta$$

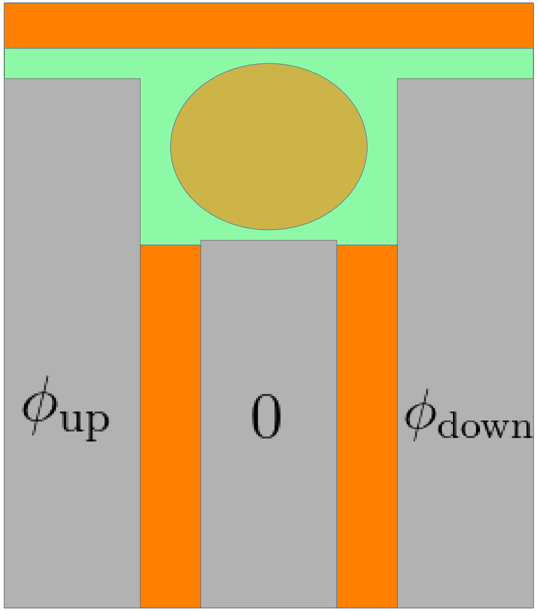
$$k_{SO}R \simeq 0.3$$

But we use bigger junctions!

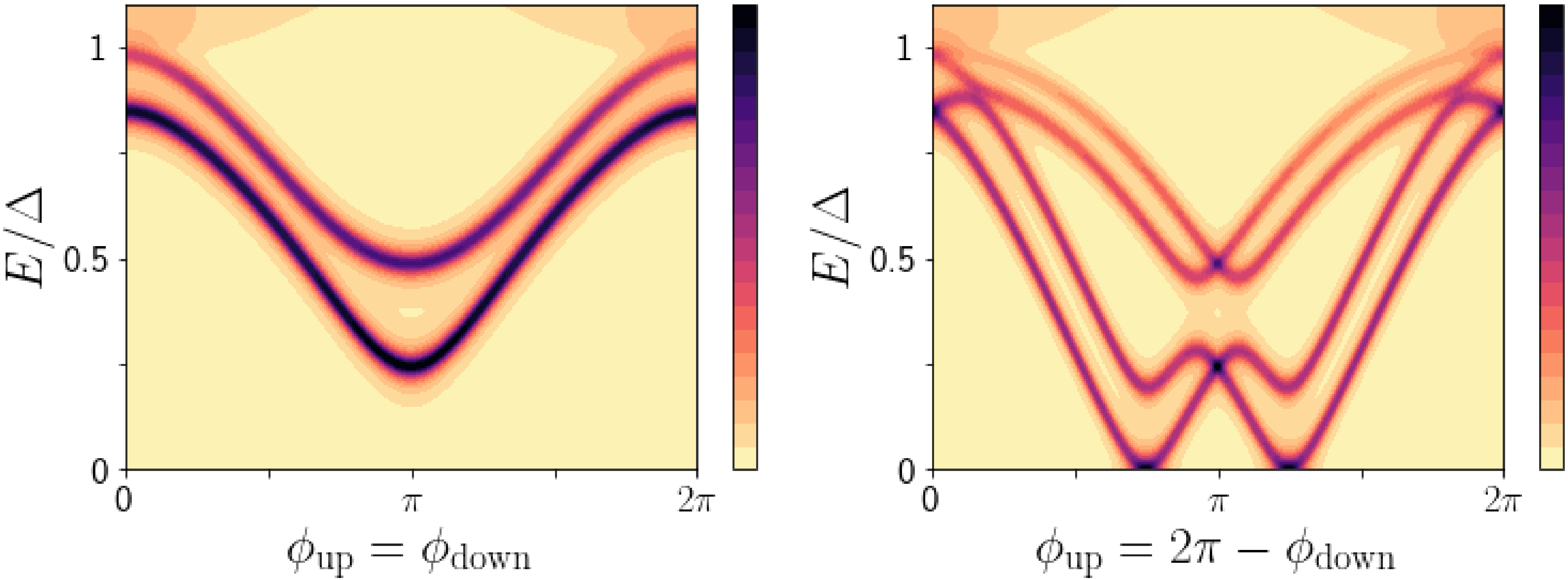
$$k_{SO}R \simeq 0.45$$

Results





Results

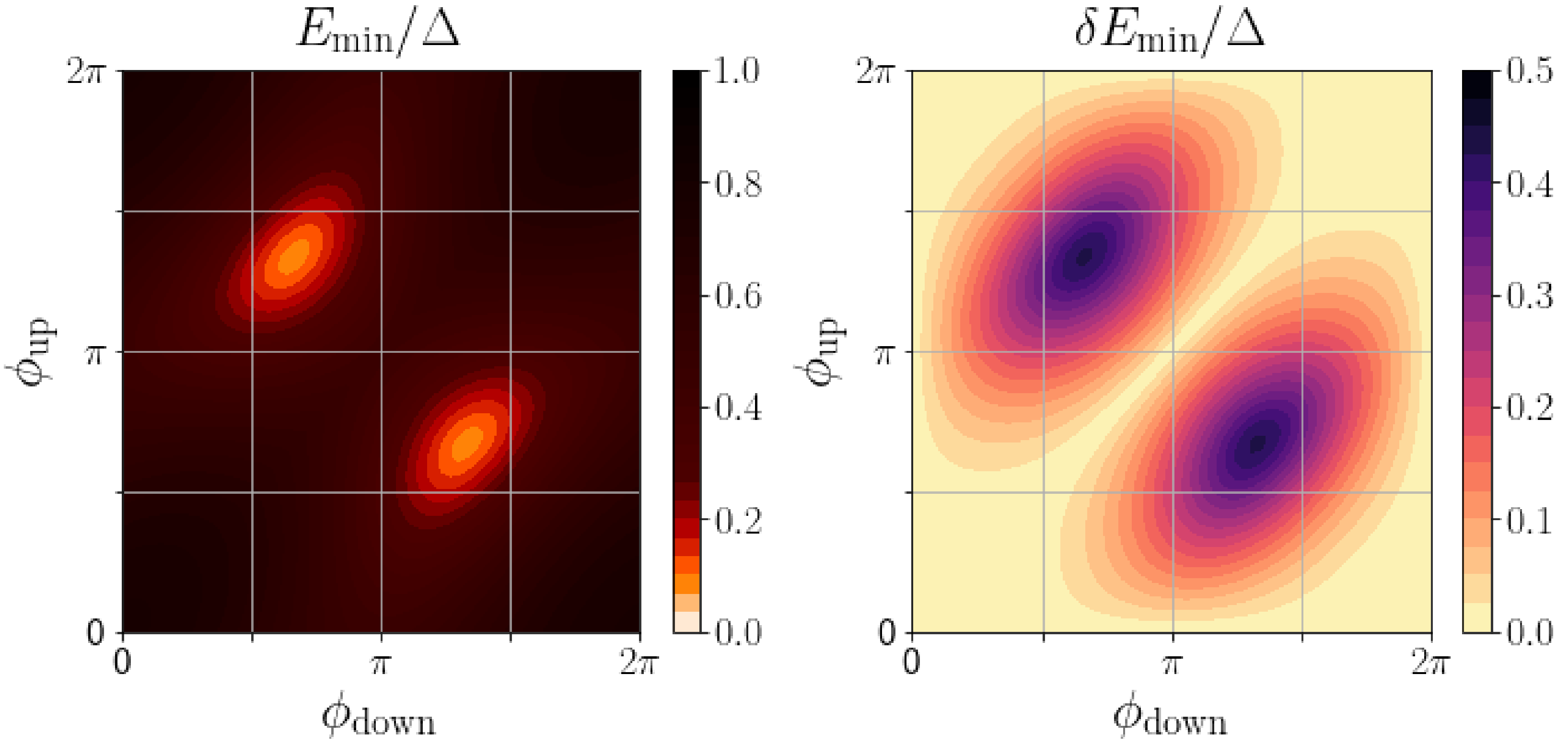


E/Δ
1
0.5
0
0
π
2π
$\phi_{\mathrm{up}} = \phi_{\mathrm{down}}$
E/Δ
1
0.5
0
0
π
2π
$\phi_{\mathrm{up}} = 2\pi - \phi_{\mathrm{down}}$

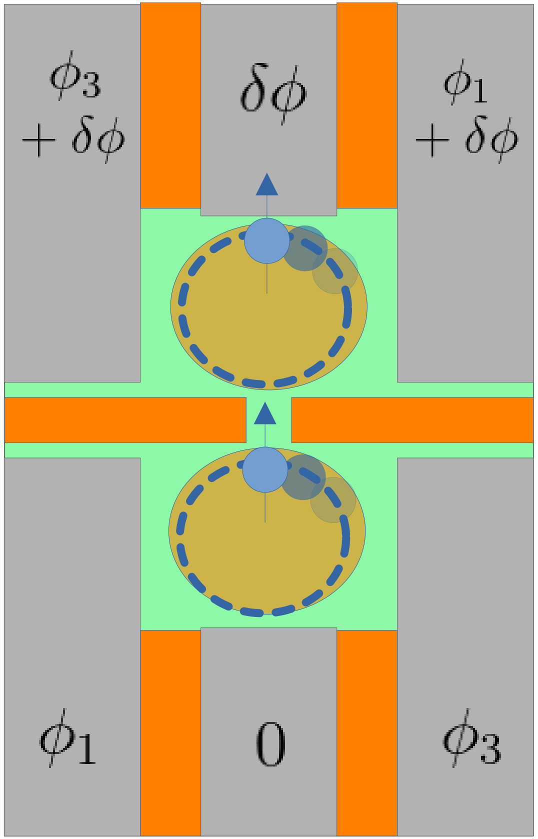
Results

$$\phi = \arctan\left(\frac{\sqrt{(x^2+3)^2 \pm 2x\sqrt{2+3x^2}}}{-1 \pm x\sqrt{2+3x^2}}\right)$$

$$x \equiv \sin(2k_{\mathrm{SO}}R)$$



Model

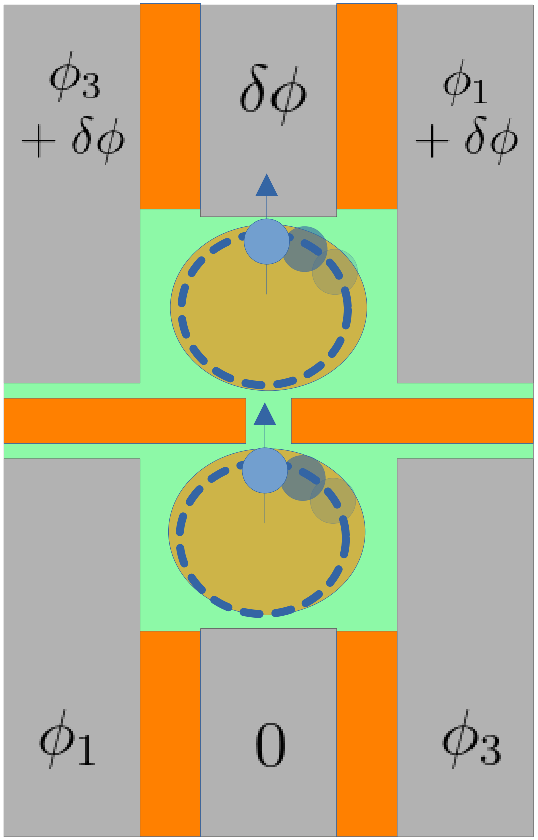


Two new (tunable) parameters between junctions:

Hopping t_{QD}

Phase difference $\delta\phi$

Model



Two new (tunable) parameters between junctions:

- Hopping t_{QD}
- Phase difference $\delta\phi$

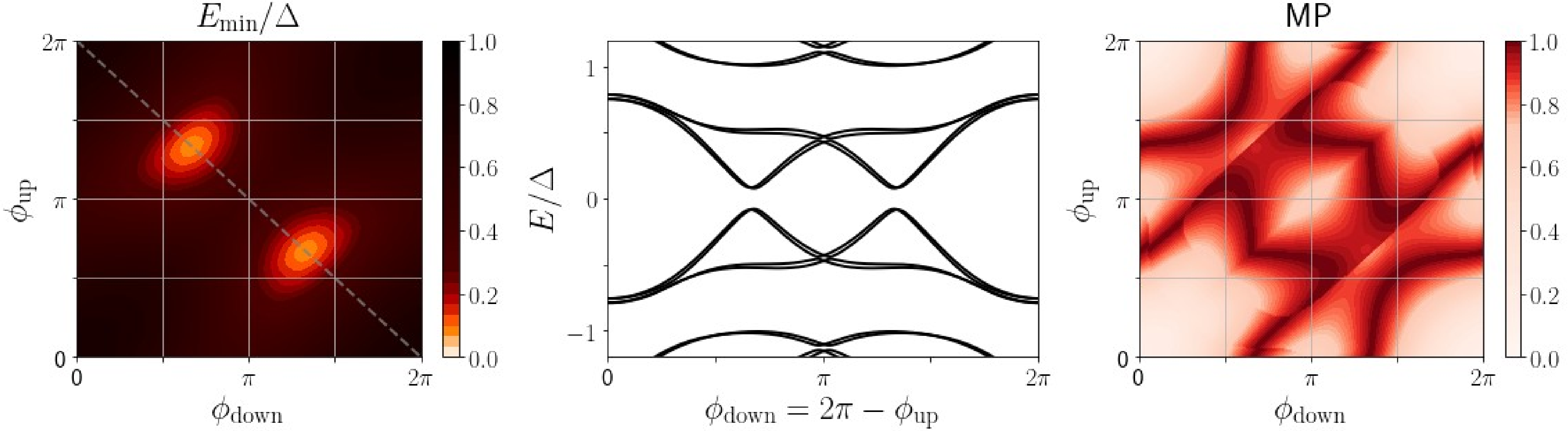
We look for zero-energy modes with MP=1.

$$\mathrm{MP}_j = \frac{\left|\sum_{\sigma,s} \langle e|\gamma_{j\sigma s}|o\rangle^2\right|}{\sum_{\sigma,s} \left|\langle e|\gamma_{j\sigma s}|o\rangle^2\right|} \qquad \mathrm{MP} = \frac{\mathrm{MP}_1 + \mathrm{MP}_2}{2}$$

Results

Weakly coupled junctions

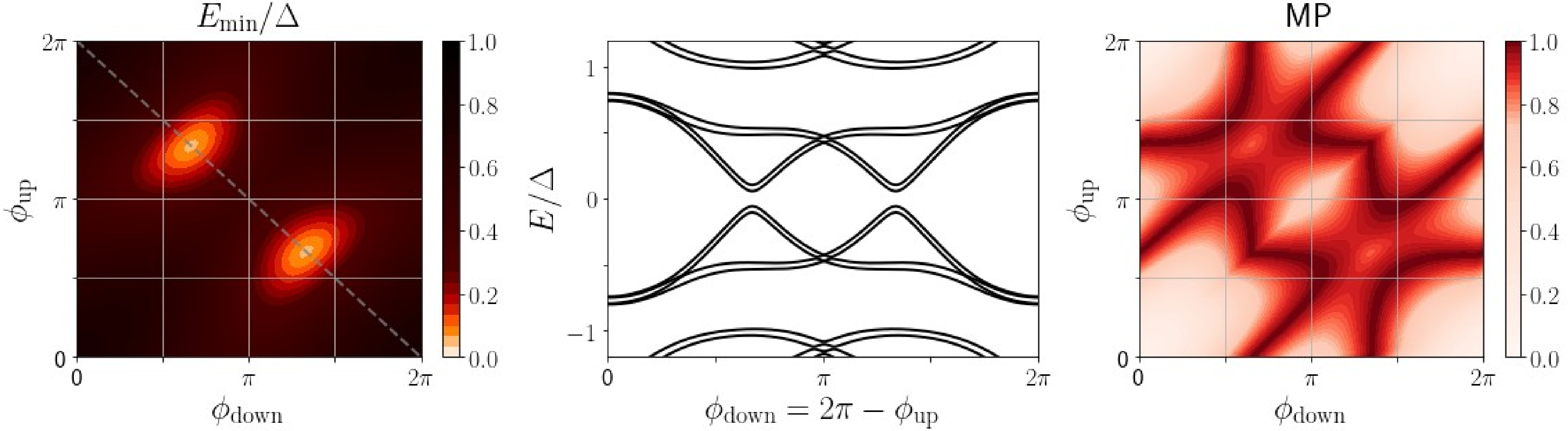
$t_{\mathrm{QD}}/t = 0.10 \quad \delta\phi = 0.00\pi$



Results

Weakly coupled junctions

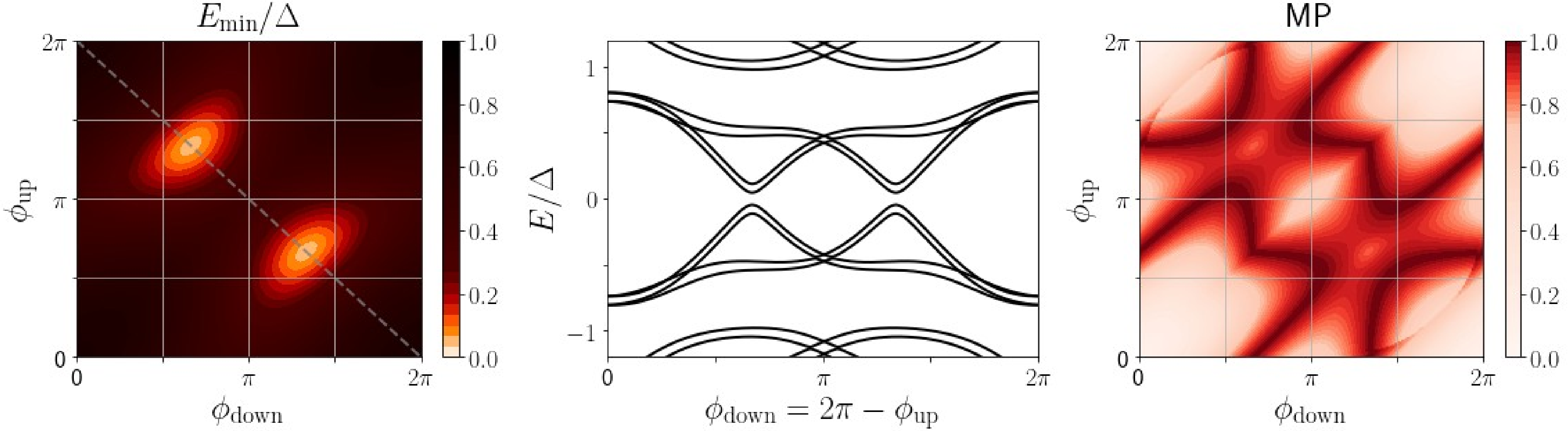
$t_{QD}/t = 0.10 \quad \delta\phi = 0.50\pi$



Results

Weakly coupled junctions

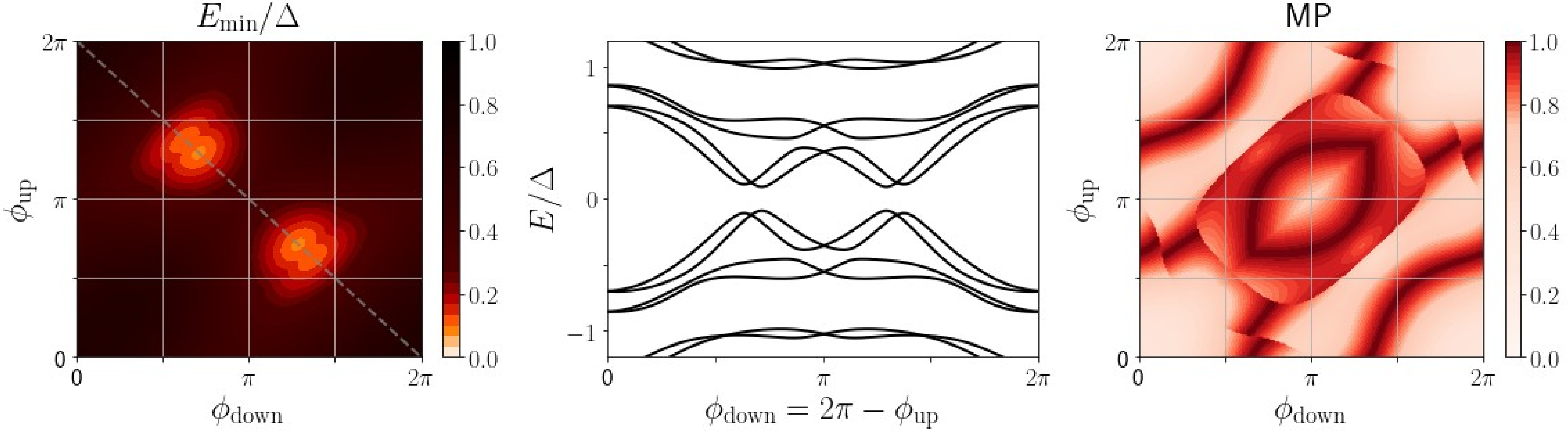
$t_{\mathrm{QD}}/t = 0.10 \quad \delta\phi = 1.00\pi$



Results

Stronger coupling

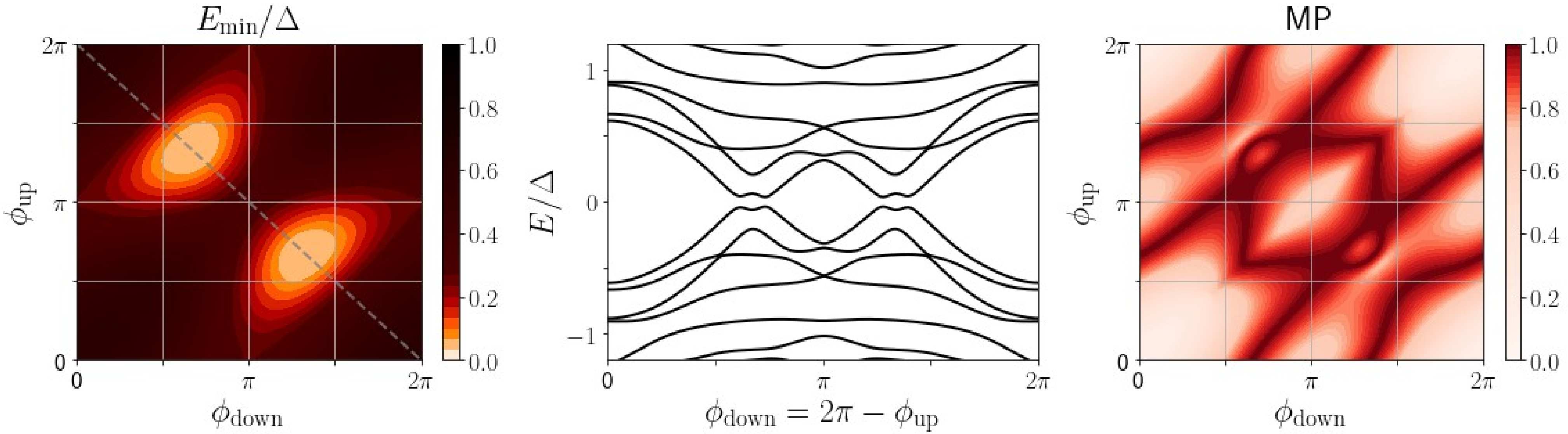
$t_{\mathrm{QD}}/t = 0.50 \quad \delta\phi = 0.00\pi$



Results

Stronger coupling

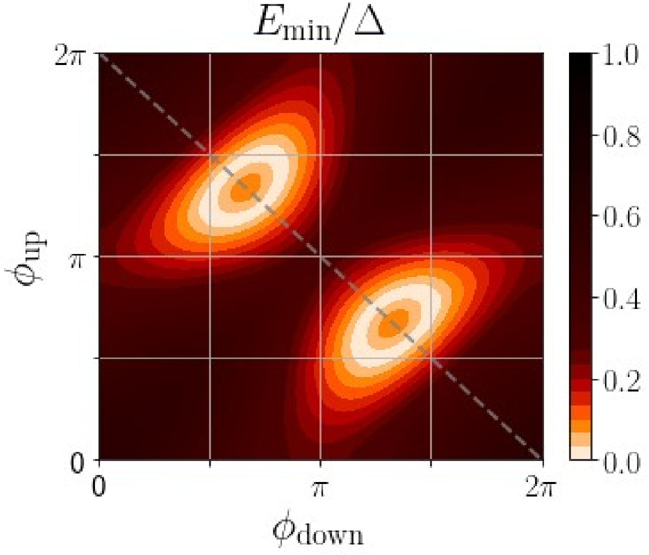
$$t_{\mathrm{QD}}/t = 0.50 \quad \delta\phi = 0.50\pi$$

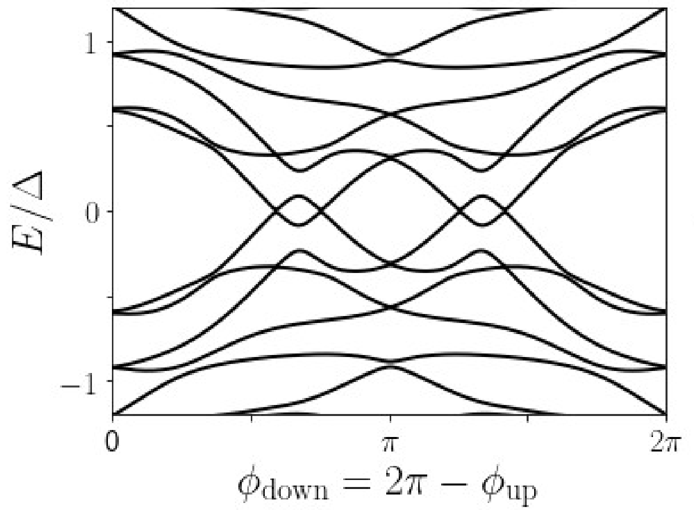


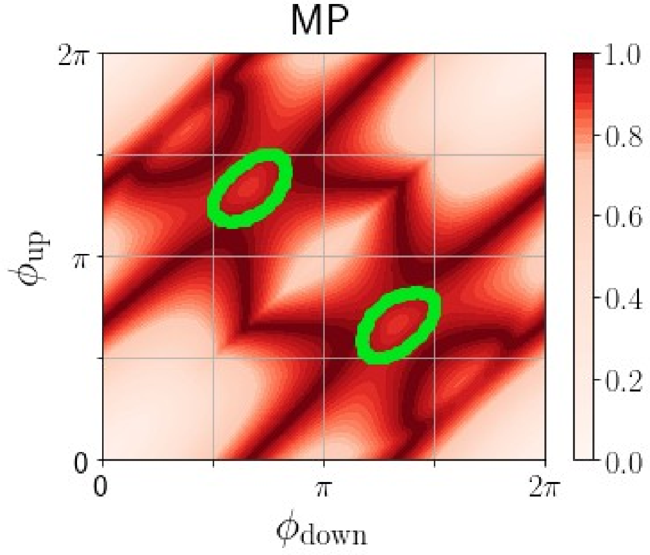
Results

Stronger coupling

$$t_{\mathrm{QD}}/t = 0.50 \qquad \delta\phi = 0.90\pi$$



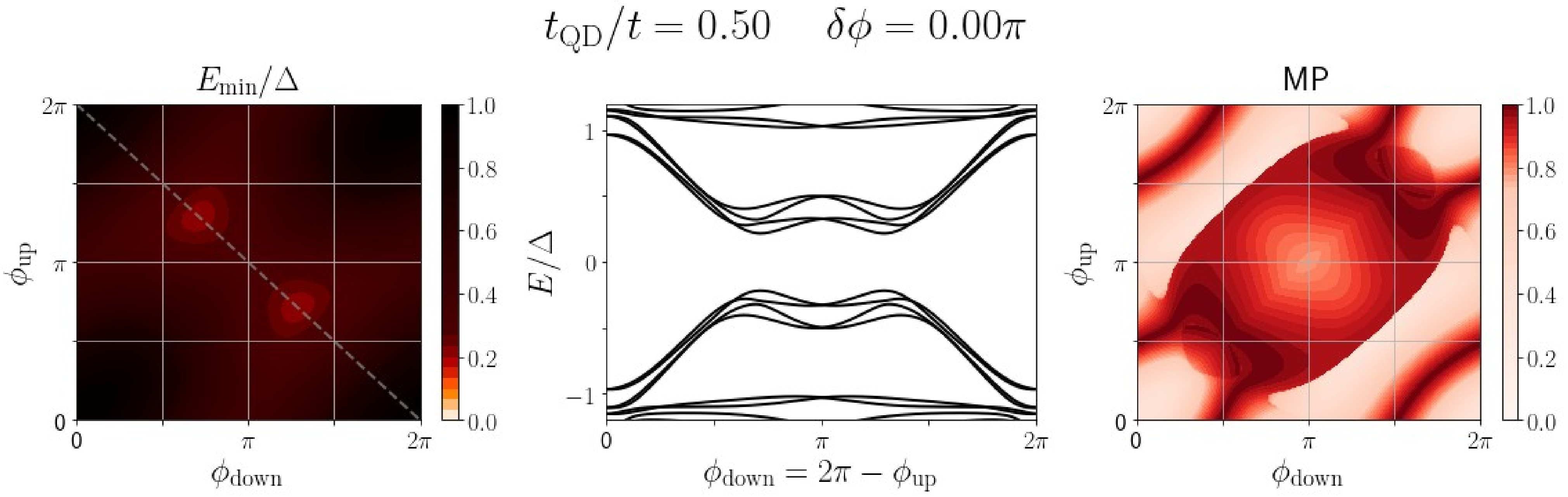




$$\phi = \arccos\left\{\frac{1}{4} \pm \frac{1}{4}\sqrt{1 + 4\left(1 + \left(\frac{t_{\mathrm{QD}}}{\Gamma\sqrt{2}}\right)^2\right)}\right\}$$

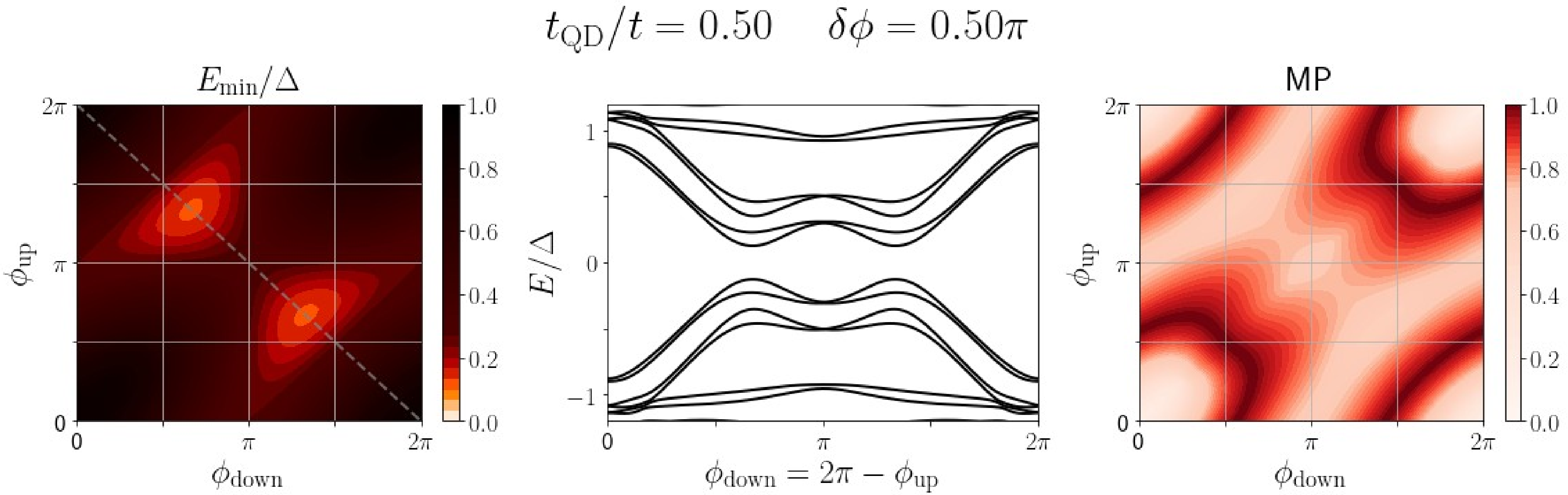
Results

Weaker SOC



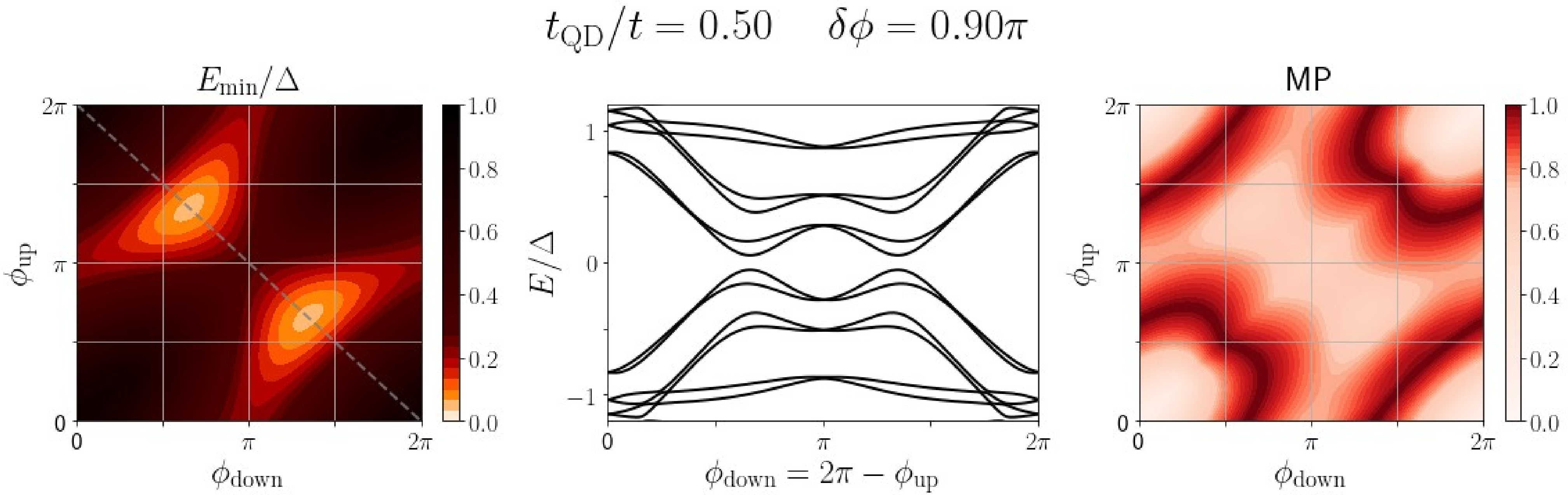
Results

Weaker SOC



Results

Weaker SOC

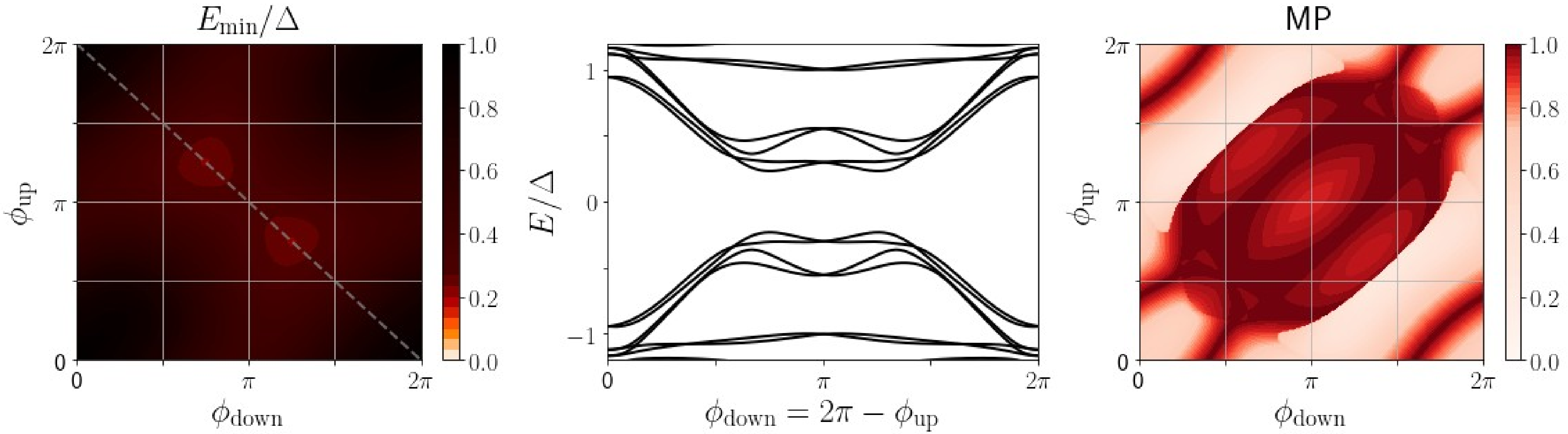


Results

Weaker SOC

Strongly coupled junctions

$t_{\mathrm{QD}}/t = 0.80 \quad \delta\phi = 0.00\pi$

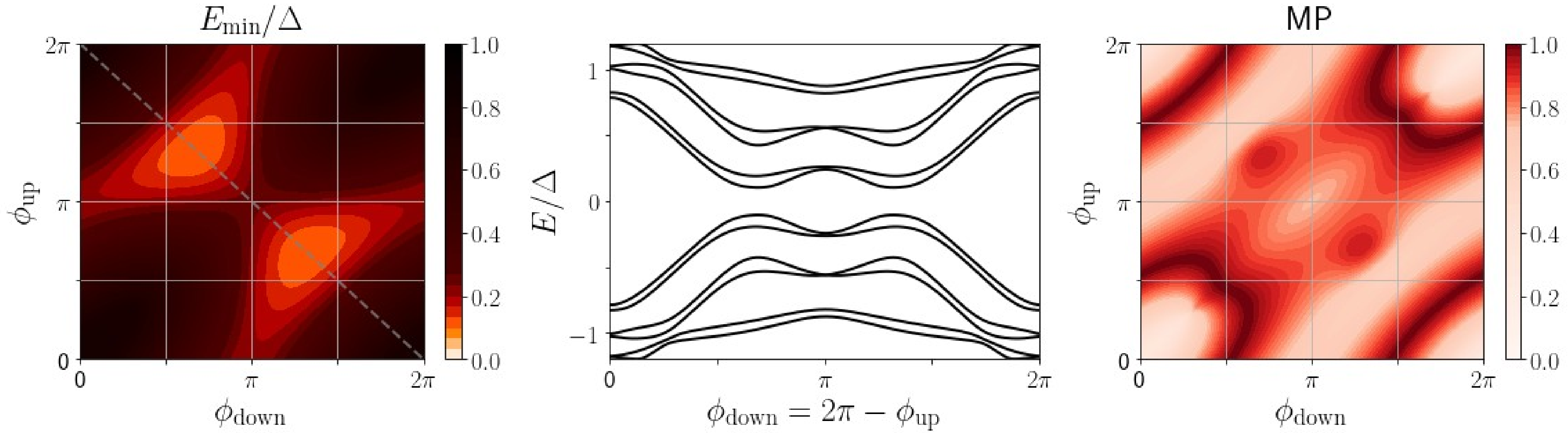


Results

Weaker SOC

Strongly coupled junctions

$$t_{\mathrm{QD}}/t = 0.80 \quad \delta\phi = 0.50\pi$$

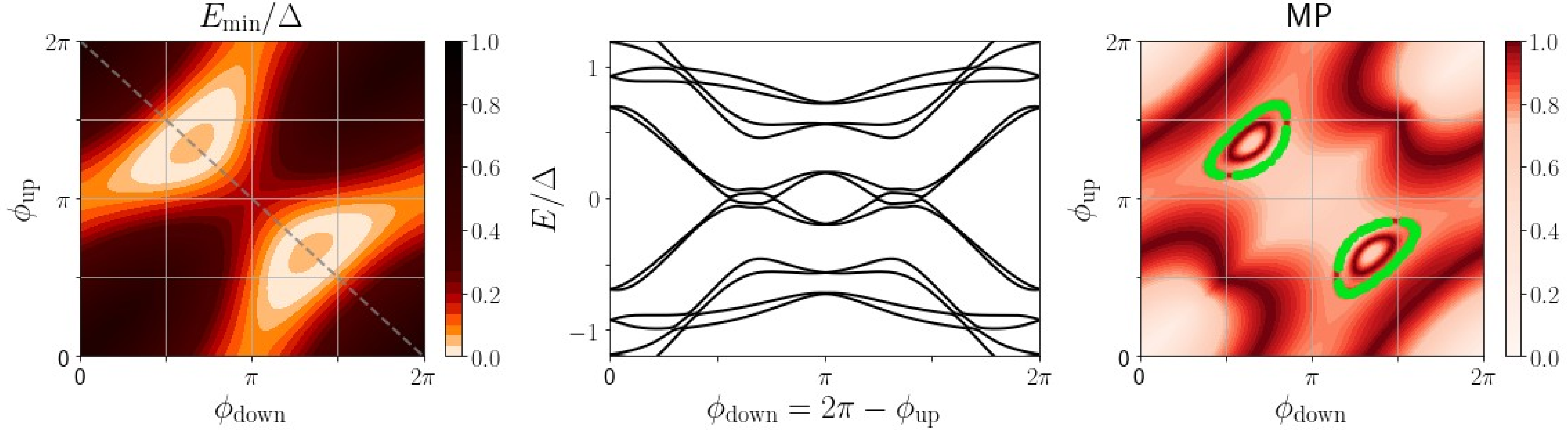


Results

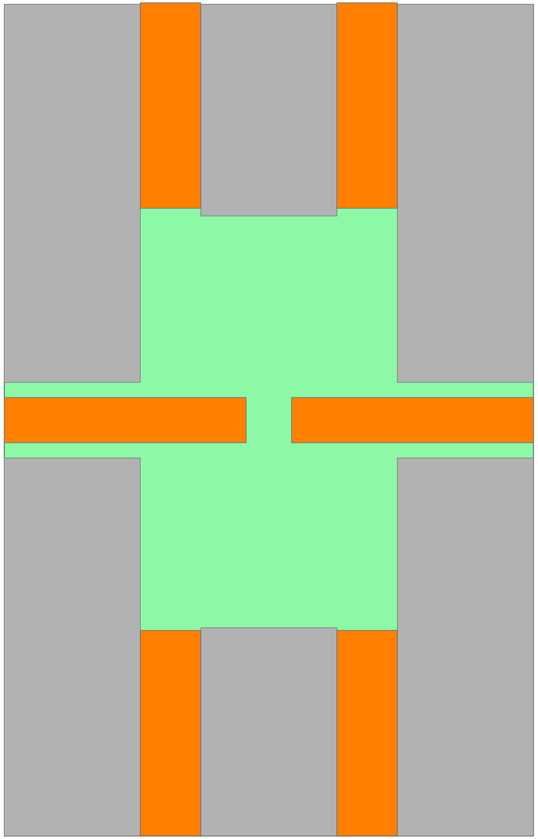
Weaker SOC

Strongly coupled junctions

$$t_{\mathrm{QD}}/t = 0.80 \quad \delta\phi = 0.95\pi$$



Results



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$$+\Delta(\vec{r})\sigma_0(\cos\phi(\vec{r})\tau_x + \sin\phi(\vec{r})\tau_y)$$

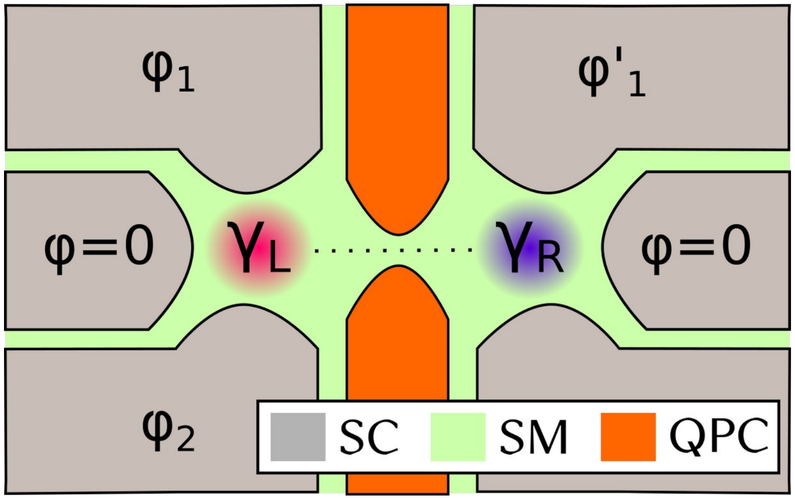
We make full-simulations of the continuum model solving the Schrödinger-Poisson equation in the entire device

We discuss the protocol to find Poor's man Majoranas

Conclusions

It is possible to create Poor's Majorana modes with superconducting phase control.

Do not exhibit larger minigaps, but there is no need of magnetic field and it is more insensitive to charge perturbations.



Collaborators

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